

Michael Atiyah

Born 22.4.1929

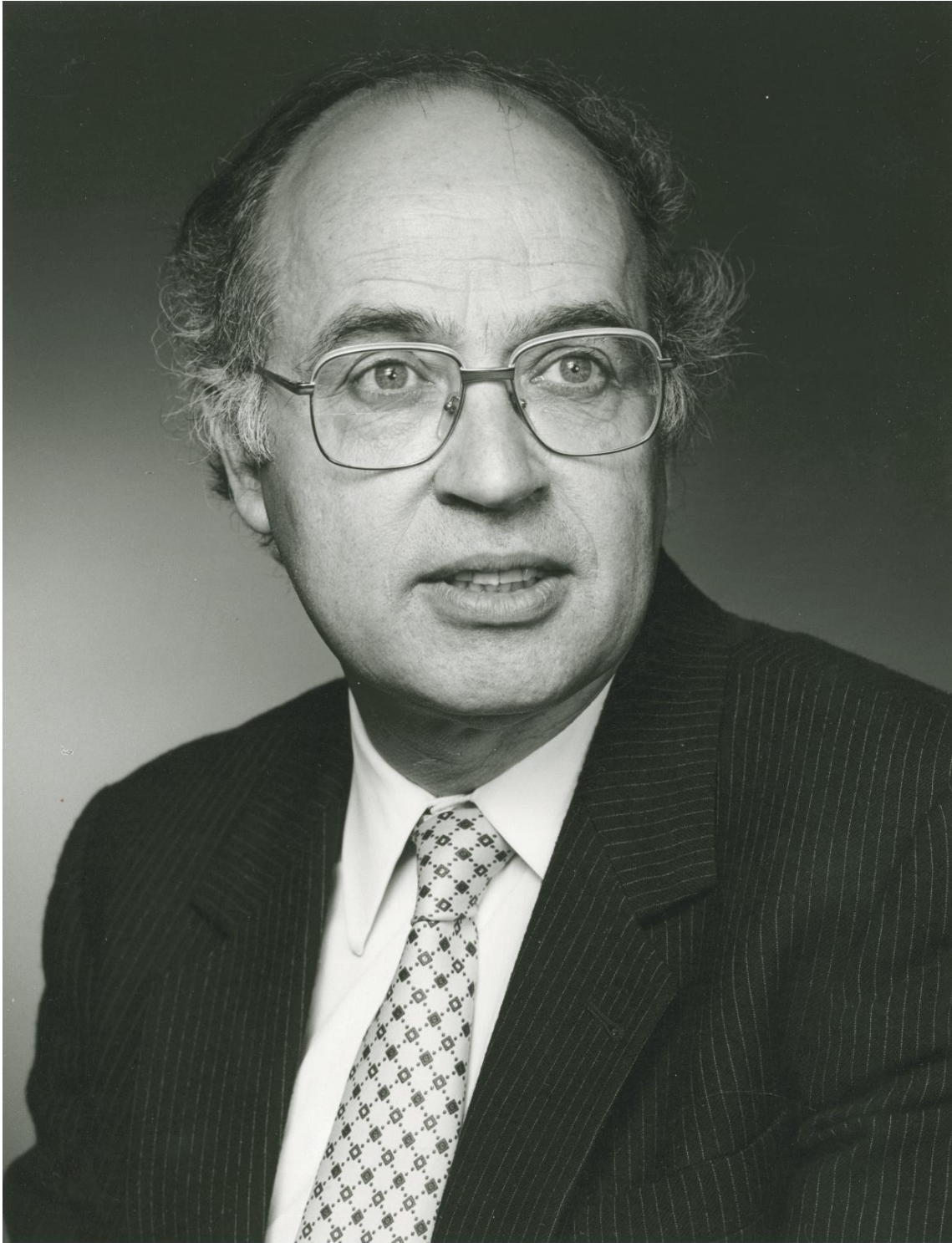
Royal Society biography by Nigel Hitchin.

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M. F. Alyek

1. Early life

Michael Atiyah was born on 22 April 1929 in Hampstead, London, the eldest of four children. His father, Edward Atiyah, came from a family of Western-oriented Lebanese Christians. Previous generations had worked for the Anglo-Egyptian administered Sudan government, and Edward was no exception. Thus Khartoum was where Michael spent many childhood years (figure 1).



Figure 1. Sudanese childhood, aged two.

Edward's career was closely associated with British culture, and, after attending Victoria College in Alexandria, he graduated with a degree in history from Brasenose College, Oxford. His subsequent life was led as a civil servant, author (including detective stories), broadcaster and promoter of the Arab cause; he became secretary of the London office of the Arab League. It was as a student in Oxford that he met Jean Levens, Michael's independent-minded Scottish mother. Edward had encountered her brother, Robert, at the Oxford Union and gradually became a frequent visitor at the family home. In 1928 they married, spent their honeymoon in Florence and named their first child after Michelangelo.

Michael's first school was the Clergy House in Khartoum, consisting of a small group of 25 pupils of all ages. When the time came to move to secondary school, the start of the Second World War narrowed his options. He spent a term in a

French school in Lebanon and then moved briefly to an Italian school in Khartoum, but survived only four days of being taught in Arabic (although this was his first spoken language) and then returned to his former school. Finally in 1941 his parents decided, with the help of a scholarship, to send him to his father's old school, Victoria College, which by that time had moved to Cairo as the original buildings had been converted into a military hospital. Here he was placed in classes two years ahead of his age (he was doing calculus at 13) and was surrounded by a stimulating group of international pupils and English teachers. His parents already recognized that mathematics was one of his strengths.



Figure 2. Victoria College, Alexandria, 1945.

From an early age, the heat of the Sudanese summer meant that the family moved away for four months, visiting relatives in England, the Lebanon, Palestine and Egypt. Travel was part of family life, and even returning to school after the summer vacation meant a four-day journey by train and Nile boat with his younger brother, Patrick. Life in Egypt was not without its dangers and aged 13 he was struck down with meningitis and only cured after a hectic rush around Cairo to find some penicillin.

In August 1944 Victoria College moved back to Alexandria (figure 2) and from there, Michael's parents decided that his mathematical ability required him to aim for Cambridge. Thus, in 1945, at the age of 16, he was sent to Manchester Grammar

School to prepare appropriately, living independently in a lodging house. His classmates, knowing only that he was from the Middle East, welcomed him with a camel drawn on the blackboard.

Having been two years ahead in Victoria College, Michael had already taken his Higher School Certificate examinations, but he was now exposed to a new level of mathematical study that required a great deal of hard work and in a competitive environment. He was taught by the chief mathematics master F. L. Heywood (an Oxford first in mathematics), who disliked textbooks and produced detailed handwritten notes describing the correct approach as he saw it. The high pressure atmosphere yielded results, with three scholarships to Trinity College, Cambridge, that year and two to St John's. At the time, Michael had little idea of which college he should apply to, but the school had a definite pecking order and Trinity was chosen for the top mathematicians. And so began his long association with the college.

Before going to Cambridge, Michael decided to spend two years doing his National Service. This could have been avoided, or postponed as most of his contemporaries did, but largely out of a sense of duty he left school in April 1947 with the aim of joining the Royal Electrical and Mechanical Engineers. However, there was a hitch—he was regarded as having dual nationality and recommended to defer until he was 21, an age at which he could choose between the two. With characteristic logic, he pointed out that his father was in fact stateless and so really he had sole British nationality. When this was accepted, he was promptly summoned to register within 24 hours and in October of that year he became a clerk in the regimental headquarters.

Looking back on his experience years later, Michael felt that it was worthwhile to be immersed in a very different social environment, his physical fitness improved with running and tennis, and he had plenty of spare time to think about mathematics. This was partly through books and problems sent on from Cambridge by his peers at school, but also through his own experiments; for example, with properties of higher dimensional polyhedra. What it did provide was an interlude that generated enormous enthusiasm for the subject when he finally arrived at Trinity, which was perhaps absent from those who had gone directly from the Sixth Form. He failed to rise through the ranks in the army, but was commended for his loud voice on the parade ground, a foretaste for his future students and collaborators!

2. Cambridge

Michael arrived in Cambridge in 1949, armed with an ex-serviceman's grant and a major scholarship from Trinity College that left him quite well provided for. His first contacts were his Trinity supervisors. One was A. S. Besicovitch FRS, a Russian analyst who had been there since 1927. He was a formidable character who would play eight games of chess simultaneously with his students. By contrast there was also the taciturn J. A. Todd FRS 1948, an algebraic geometer whose book he had already studied and enjoyed.

Perhaps more important were his fellow Trinity students with whom he would engage in mathematical discussions and compete in exams. These included J. F. (Frank) Adams (FRS 1964), John Polkinghorne (FRS 1974), Ian Macdonald (FRS 1979), the statistician John Aitchison and James Mackay (later Baron Mackay of Clashfern, Lord High Chancellor of Great Britain 1987–1997). Mackay recalls as a student sitting on the pillion of a motorbike going to Oxford to visit Michael's brother, Patrick. They remained lifelong friends—in later years Michael persuaded him as Lord Chancellor to release the files of Operation Epsilon, the surveillance of detained German scientists who had worked on the Nazi nuclear programme.

College life was quite spartan in that postwar period, with food rationing, inadequate heating and limited washing facilities, but there were many activities to join in, such as chess and squash and the students' mathematical society, the Archimedean. In fact 'at a meeting of the Archimedean I came across a little man sitting on a table with his legs not even reaching the ground' was how Lily Brown recalled first meeting Michael. The strong-minded Scot, the first in her family to go to university, had read mathematics in Edinburgh with Mackay and then moved to Girton College, Cambridge, to complete the Tripos. That first encounter with Michael led to a long courtship and finally marriage in 1955.

For the first two years of the Tripos, Michael and John Polkinghorne worked over past papers intensively, which led them to come top of the list in the results, but by the time of Part 3 Michael in particular was beginning to focus his interests and becoming less concerned with examination results. Geometry was central to these interests. This was a period in Cambridge of transition between the (to Michael) attractive classical geometric approach and the more rigorous one based on algebra. The two relevant texts were Todd's book, *Projective and analytical geometry* (Todd 1947), which included the algebra reluctantly but necessarily, and *Methods of algebraic geometry* by W. V. D. Hodge FRS and D. Pedoe (Hodge & Pedoe 1952), which accepted it face-on, and Michael had to absorb both. His first paper, produced in his second undergraduate year, is written in the purely classical vein, but this was not to last.

When the time came to enrol for a PhD, students had to choose a supervisor, and Michael, wanting to pursue research in geometry, had the choice of Todd or Hodge. He had been the star pupil for Todd at Trinity, but, perhaps to Todd's disappointment, chose Hodge as having the bigger international reputation. Hodge, unusually, had as many as four students that year, one of whom was Roger Penrose (FRS 1972). Hodge's research in harmonic integrals was an early contribution to

what became a whole new aspect of geometry, mixing topological, differential and algebraic techniques. It was generally thought to be hard, which perhaps put off other students from working with him, but Michael had attended lectures on algebraic topology by P. J. Hilton and S. Wylie, had some basic Riemannian geometry and was reasonably well prepared.

His first task was to understand characteristic classes of vector bundles, following S.-S. Chern (ForMemRS 1985) and C. B. Allendoerfer, but quite rapidly in his first year, Michael latched on to the new techniques of sheaf theory developed by Henri Cartan (ForMemRS 1971) and Jean-Pierre Serre (ForMemRS 1974) in Paris. Reading the latest *Comptes Rendus* notes in the library, he rapidly absorbed the new techniques and by the end of his first year produced a prize-winning essay on ruled surfaces via the theory of holomorphic vector bundles, stimulated by both lectures of A. Weil (ForMemRS 1966) and discussions with the visiting American mathematician, N. S. Hawley. Ironically, the discovery of a mistake in one of Hawley's papers was a pivotal event for this work, an issue that his supervisor recommended he tone down a little in the resulting paper, which is written in a thoroughly modern style.



Figure 3. Research fellow, Trinity College, 1954.

Watching a number of his contemporaries leave after a year, the prize essay gave him enough confidence to proceed further, but not without self-doubts. He tempered

the loneliness of mathematical research by going to lectures on architecture and archaeology, which he found interesting enough but confirmed his belief that mathematical research was harder and more worthwhile. A key event in the following years was attending the 1954 International Congress of Mathematicians in Amsterdam, where the award of Fields Medals to Serre and K. Kodaira offered further verification that his research field was of great importance.

By the end of his second year as a PhD student Michael had his prize work and a paper with Hodge on integrals of the second kind, and on this basis was elected a Fellow of Trinity (figure 3), but, satisfying as this was, Cambridge offered few new ideas in his third year or opportunities for visitors. During 1953–1954, Hodge had been to a conference in Princeton and explained what Michael had been working on and, as a consequence, D. C. Spencer invited him to the university, but then the award of a Commonwealth Fellowship enabled him to travel and stay longer, at the Institute for Advanced Study instead of Princeton University.

His domestic life, however, had changed. On his election as a Fellow, Michael and Lily became engaged and she, being a year ahead of him, had by 1955 graduated with a PhD under the supervision of Mary Cartwright FRS and was employed as a lecturer at Bedford College, London. She gave up her job, they married in Edinburgh (with James Mackay as best man) and set off for Princeton.

3. Princeton

The stay in Princeton was a key period in Michael's mathematical life. It was not only the presence of Kodaira and Serre, the recent Fields medallists, but also a stream of visitors bringing new ideas and perspectives. It was here that Michael met his three future prime collaborators and personal friends, Raoul Bott (ForMemRS 2005), Isadore Singer and Friedrich Hirzebruch (ForMemRS 1994), the latter he had briefly encountered before when Hodge invited them to talk in Cambridge. Bott lectured on Morse theory, Kodaira on sheaf theory, currents and algebraic geometry, and Hirzebruch on characteristic classes. Three days a week, a carload of mathematicians including Michael, Bott and Singer eagerly travelled to the university to hear these seminars.

Michael spoke three times in Serre's seminar on analytic vector bundles, but was initially taken aback by the demand from a Bourbaki-influenced audience demanding precision and clarity, quite removed from the gentlemanly atmosphere in Cambridge. The influence of this seminar manifested itself in Michael's papers published at the time on the sheaf-theoretical description of characteristic classes and on holomorphic vector bundles on elliptic curves. These were topics he spoke on as he travelled extensively for three months around America, part of the conditions of a Commonwealth Fellowship. He met Chern in Chicago and spent a month in Mexico, where, in 1956, S. Lefschetz (ForMemRS 1961) had organized an international symposium on topology in Mexico City. Ostensibly concerning topology, it is noticeable from the titles of talks that fibre bundles, characteristic classes and the Riemann–Roch theorem were already changing the face of that subject.

It was a period when various strands of mathematics, perhaps not apparent at the time, were coming together to form a new discipline. Paris was a significant source, but, when transported to Princeton, the French algebraic geometry merged with differential topology to form a much larger framework. And it was not the permanent members at the Institute who were leading this movement, but the young visitors. It was a real melting pot of ideas and a formative experience for Michael.

4. From America to Cambridge

At the beginning of 1957 Michael returned to Cambridge as a university lecturer, and then the following year was also appointed as a teaching fellow at Pembroke College. He was expected to bring back some of the excitement of doing mathematics in the United States and, somewhat surprisingly for a young don, the professors gave him a free hand.

Those who knew him would understand this—he was an ebullient character, eager to convey his thoughts and ideas in a strong voice, which would become familiar across many a departmental common room. It was not just his own work, but any new piece of mathematics he found interesting could be the subject of his energetic exposition. Students were not exempt—one baffled Cambridge undergraduate recalled going to his supervisor, Peter Swinnerton-Dyer (FRS 1967), to be told ‘Dr Atiyah treats the syllabus with the utmost contempt’. It was the scope of the lecture rather than his presentation—throughout his whole career he was famous for his inspiring seminars and lectures linking different themes together. Reconstructing the argument later was usually a different matter.

So, in Cambridge, he began to organize colloquia. There was still little opportunity for inviting outside speakers, but professors finding out what each other was doing was itself a novelty. At the same time he was recounting the ideas he had acquired in the United States to Hodge and others, like E. C. (Christopher) Zeeman (FRS 1975) who too had recently returned from America. Hodge himself was becoming increasingly occupied with other activities—Master of Pembroke and Physical Secretary of the Royal Society—and he handed over to Michael many other departmental activities and some of his graduate students too, such as Rolf Schwarzenberger and Ian Porteous.



Figure 4. With Grothendieck, Bonn, 1958 (image from the Hirzebruch family collection; copyright unknown).

At this time, in 1957, the Bonn *Arbeitstagung* (figures 4 and 5) began and Michael became a regular attendee, forging closer links with Hirzebruch, who, at a young age, had been appointed professor there. It was a very active period for topology, with Milnor's work on manifold classification, Thom's on cobordism theory and Hirzebruch showing the links with algebraic geometry. The topics were not chosen in advance, so that the meeting became a shop window for the latest results of the likes of Serre, Jacques Tits, Armand Borel and Alexandre Grothendieck as well as Michael's former Princeton associates. Grothendieck in particular took centre stage at the beginning—the others sat back and learned from it (figure 4).

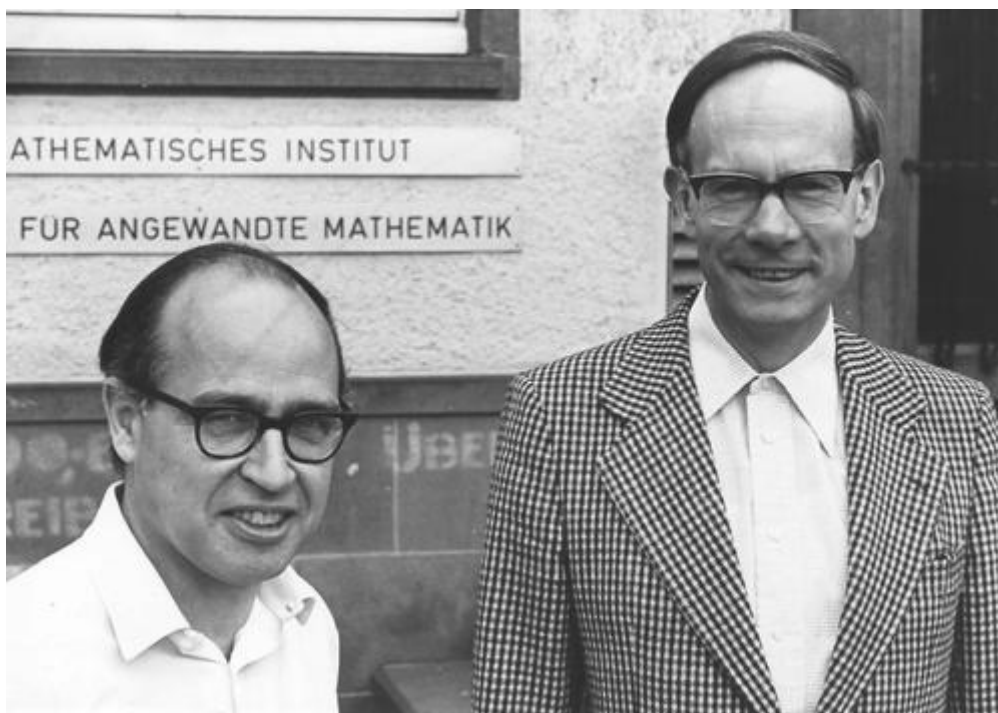


Figure 5. With Friedrich Hirzebruch at the *Arbeitstagung*, Bonn, in 1977. (Taken by Konrad Jacobs, Erlangen. Copyright: Archives of the Mathematisches Forschungsinstitut Oberwolfach.)

Cambridge also saw another development in Michael's life—the birth of sons John (1957) and David (1958).

5. K -theory

Michael was not by nature a theory-builder, but sometimes it became a necessity. One of his mathematical achievements, begun in 1959 collaborating with Hirzebruch during a brief return to Princeton, was the invention of K -theory. This was an adaptation into a new environment—algebraic topology—of Grothendieck's formulation of the Riemann–Roch theorem in algebraic geometry, which Michael had been exposed to in the *Arbeitstagung*. Its introduction was part of a general drift of ideas from algebraic geometry to topology, a big step in this direction having been taken by Hirzebruch in his book, *Neue topologische Methoden in der algebraische Geometrie* (Hirzebruch 1956).

Although not by training an algebraic topologist, Michael began to involve himself more and more in the subject. He observed that a number of problems in topology, in particular some posed by Ioan James (FRS 1968), would become much easier to prove if one could produce a cohomology theory based on topological vector bundles. Moreover, the integrality results of characteristic classes in Hirzebruch's work would become clearer. To construct a coherent theory, new features not present in Grothendieck's work, such as the odd degree case and the real theory, were needed. Its properties required the results of Bott on the periodicity of the homotopy groups of the unitary and orthogonal groups that Michael had heard about in Princeton. The resulting theory, K -theory, subsequently became the natural vehicle for the index theorem, but within the realm of algebraic topology it had numerous achievements. One of them was, proved together with Adams, a simple proof of the classical Hopf invariant problem concerning the degrees of maps between spheres.

For the next three years or so, Michael's papers were devoted to applications of K -theory, and he gave an invited talk, 'The Grothendieck ring in geometry and topology', at the 1962 International Congress in Stockholm, driving across Norway with his family and mother (a stopover in a log cabin was the impetus for buying one in Perthshire 15 years later). However, during this period there was another change in his life.

6. Oxford

While in Cambridge, Michael's work attracted the attention of the topologist J. H. C. (Henry) Whitehead FRS in Oxford, and he went there to give talks, but in 1960 Whitehead died suddenly. The Waynflete Chair, which he had occupied, became vacant and Michael, at the age of 31, applied for it. In the event, it went to the more senior Graham Higman FRS, a group theorist who was already there as a reader. So Michael successfully applied for this readership instead of the chair, and took it up in 1961. He chose not to engage in college teaching and could then spend more time doing research.

Mathematics at the time was not one of Oxford's strengths, although there was at least a central building—the Mathematical Institute. The closest interest for Michael was topology, represented by Ioan James, also a reader, but Michael, who became a Fellow of the young St Catherine's College, began instead to acquire a group of graduate students, many sent on from Cambridge by Hodge. Graeme Segal (FRS 1982) was one of them. He recalls Michael's advice at the time: 'People are always asking me to give them a problem, but they don't mean that; what they want is a method. I give my students a method.' By that time he had acquired a group of six students and 'a little cloud of de facto students offically belonging to others'. To compensate for the lack of active geometers, Michael spent any available time going to Harvard or Princeton or inviting Bott or Singer to Oxford.

In 1963 the number theorist E. C. Titchmarsh FRS died, and the Savilian Professorship of Geometry that he had held was offered to Michael, with a fellowship at New College. Since Titchmarsh's predecessor was G. H. Hardy FRS, Michael was the first geometer to hold that chair for some time. Michael and Lily bought a house in Headington, and extended it in 1966. The long garden was not only a chance for Michael to indulge in one of his hobbies (music and hill-walking being others), but also a place to pace up and down with his mathematical thoughts. A third son, Robin, was born in 1963.

7. The Index Theorem

The Atiyah–Singer index theorem is Michael's best known result. It won its authors the Abel Prize in 2004 (figure 6) and papers on the topic occupy two volumes of Michael's collected works.



Figure 6. The Abel Prize 2004. (Image taken by Knut Falch © Scanpix Norway/PA Images.)

Its origins lie again in the osmosis of concepts from algebraic geometry into topology. The Riemann–Roch theorem in its modern formulation concerns the alternating sum of dimensions of sheaf cohomology groups

$$\sum_{i=0}^n (-1)^i \dim H^i(M, V) = \sum_{i=0}^n (-1)^i \dim H^i(M, V)$$

and is fundamental for many processes in algebraic geometry. In Hirzebruch's early work it is expressed as a combination of characteristic classes—a topological invariant. Collecting together the even degrees and the odd ones, this is a difference of the dimensions of two vector spaces. Hirzebruch's methods also applied to the signature of a differentiable manifold, which, through the theory of Hodge's harmonic forms, is also a difference of dimensions. Both cases can be expressed as the difference in the dimensions of the null spaces of two associated differential operators on a compact manifold. This is the setting of the index theorem.

Through careful manipulations, Hirzebruch had been showing at the *Arbeitsstagen* how certain combinations of characteristic numbers were always integers, and this also demanded an explanation in terms of dimensions. These observations constituted an answer in search of a problem, and that general problem was to determine the difference in dimensions of the null space of an elliptic operator D and its adjoint D^* , the so-called *index* of D :

$$\text{ind } D = \dim \ker D - \dim \ker D^* \quad \text{ind } D = \dim \ker \{f_0\} D - \dim \ker \{f_0\} D^*.$$

The Riemann–Roch theorem was a very special case.

The fact that the index was a topological invariant was known to the Russian analysts, but not a precise formula. In the spring of 1962, Singer came to spend a sabbatical term in Oxford and posed a specific question about the so-called $A^{\wedge}A^{\wedge}$ -genus that was an integer for a spin manifold, according to a result of Hirzebruch. Once they had realized, without a proof, that this must be the index of the Dirac operator in its Riemannian expression, they had enough information to begin the programme, which took 18 months before a first proof was found. Singer's early background in mathematics was in the area of analysis and he brought that expertise to the table to complement the topology and algebraic geometry that were Michael's forte.

The period that followed this first successful attack on the index theorem led on to a large number of papers exploring generalizations and ramifications of the original idea. Many of these involved group actions, the origins of which lay in walks along dusty paths at Woods Hole near Cape Cod with Raoul Bott. It was a conference in algebraic geometry and number theory, and what started out as a conjecture of G. Shimura on automorphisms of algebraic curves blossomed into a general theory of Lefschetz fixed point formulas for elliptic complexes, which included H. Weyl's famous character formula as a special case.

As the work progressed, the language of K -theory came to be seen as the appropriate framework for index problems, and in 1968 came the first of the five papers in *Annals of Mathematics* with Singer which gave proofs of the index theorem in many different contexts with many different applications.

By this time, Michael had been elected a Fellow of the Royal Society in 1962 and won a Fields Medal at the 1966 Moscow International Congress for his work on K -theory and the index theorem. The year following the Fields Medal, he paid his third visit to the Institute in Princeton, and, as with other Fields medallists, was approached to see if he would like to go there as a permanent member. After some deliberation back in Oxford, in 1969 he decided to move to a place that had good memories and better facilities for interaction. He did not fully leave Oxford, for the Institute term finished in April and the family returned to their house and Trinity term each year.

8. Back to Princeton

The index theorem continued to be a subject of research in Princeton, largely through the presence of a young Indian mathematician, V. K. Patodi, who brought with him a new method of proof.

Even after the first announcement, the original proof of the theorem was felt not to be the last word. It was not, in Michael's words, 'A proof consistent with the elegance of the problem'. It consisted first of analytical work showing that the index was a particular type of invariant in algebraic topology, namely a *cobordism* invariant—a weaker notion than topological equivalence. To prove the theorem it was then enough to check it on generators of the cobordism ring, a trick that Hirzebruch had used earlier. This had limitations for the future development of the theorem, and the more satisfactory proofs that appeared in the *Annals* involved *K*-theory and were essentially modelled on Grothendieck's approach, but replacing the properties of sheaves by those of pseudo-differential operators.

Following on from this second proof, Patodi, building on the work of Singer and H. McKean, came up with an approach using the heat kernel, formally $\text{tr exp}(-tDD^*) - \text{tr exp}(-tD^*D)$. Since the eigenspaces of DD^* and D^*D for non-zero eigenvectors are interchanged by D , the index is the difference

$$\text{tr exp}(-tD^*D) - \text{tr exp}(-tDD^*)$$

but there is an asymptotic expression as t tends to zero, which involves integrals of locally-defined terms. Patodi had deftly manipulated these to prove the index theorem in special cases. The locality of the approach offered the opportunity to develop an index theory for manifolds with boundary and a means of explaining work that Hirzebruch had been carrying out on signature defects, which yielded number-theoretic Dedekind sums. This required a more conceptual approach to Patodi's 'miraculous cancellations', and finally gave rise to a new type of global boundary value problem and a real-valued index, the *eta-function*. This later came to play a significant role in the physicists' theory of anomalies and determinants.

While resolving the problems involved in understanding these new issues, Michael made another decision. Despite the heady atmosphere of research in Princeton, he, like others before and after him, decided to leave the Institute after three years. Partially this was for family reasons—Lily was unable to teach in America, and they decided that they preferred a British education for the children. Michael had been offered a Royal Society Research Professorship for his return to the UK. It could have been held anywhere, but he chose Oxford.

9. Back to Oxford

Back in Oxford in 1972, doing research full time without undergraduate teaching, Michael took on graduate students. In Princeton this was hardly possible, the young people around him being postdocs. He was also freer to travel and collaborate, and after each visit he brought back new results and concepts, expounding them in a weekly seminar that attracted much attention, and not just in Oxford. This alternative channelling of his ideas proved to be very fruitful.

At the same time that Michael returned to Oxford, his former colleague as a graduate student in Cambridge, Roger Penrose, came to take up the Rouse Ball Chair in Mathematics (figure 7). Penrose and his students were working out the consequences of his twistor-theoretic approach to the equations of mathematical physics, and, while Michael may have been less at home with the physical motivation, they were nevertheless on the same wavelength when it came to the geometry of the Klein quadric, the cornerstone of twistor theory, which they had both learned from Todd's book all those years ago. The first fruit of the interaction was Michael's recognition that the sheaf theory he had first learned as a brand new subject while they were research students together was the appropriate language in which to describe the contour integral solution of zero rest-mass field equations that Penrose was working on. This provided a ready-made reservoir of sophisticated techniques to apply to these linear equations, which was eagerly assimilated by Penrose's students. Somewhat later, similar ideas would have even more remarkable repercussions.



Figure 7. With James Eells and Roger Penrose, Durham, 1982. (Photography by Dirk Ferus. Source: Archives of the Mathematisches Forschungsinstitut Oberwolfach.)

In early 1977, Singer paid an extended visit to Oxford and spoke in a series of seminars about what he had learned in MIT of the physicists' work on *instantons*—self-dual solutions of the Yang–Mills equations on Euclidean 4-space. For the rest of the term each weekly seminar was devoted to this topic. Two developments occurred at this time.

The first was the work of Richard Ward (FRS 2005), a student of Penrose, who had shown using twistor theory that a complex solution of the self-duality equations arose from the data of a holomorphic vector bundle on projective 3-space. By chance, Michael had attended the mathematical physics seminar in which Ward had spoken, and rapidly saw how the Euclidean version of the correspondence worked, providing a link to algebraic geometry.

The second development was that the index theorem could be put to use to actually calculate the dimension of the moduli space of instantons—all solutions modulo gauge equivalence. The dimensionality of the moduli space coincided with a dimension that W. Barth had calculated for the moduli of certain stable holomorphic bundles on projective space, and this itself was based on a very concrete construction that G. Horrocks had spoken about the previous summer. Tying all the threads together, with some differential geometric vanishing theorems, gave the final outcome in November 1977: a construction of all instantons using just finite-dimensional matrices. At the same time that Michael and the author of this memoir had found this, we heard from Y. Manin that he and his student, V. G. Drinfeld, who had been following the Oxford developments at a distance in Moscow, had independently derived the same result, which was subsequently known as the ADHM (Atiyah, Drinfeld, Hitchin, Manin) construction.

This piece of work was influential in different ways. First, it provided a further movement of ideas from algebraic geometry into differential topology, this time, unlike Riemann–Roch or the index theorem, involving non-linear differential equations. Moduli spaces had traditionally been introduced studying families of algebraic varieties or sheaves, but now there were important examples in differential geometry. In this new world the index theorem played a supporting role in describing the tangent space, a linear approximation to the non-linear moduli space. This had profound consequences when Michael's student, Simon Donaldson (FRS 1986), took the Yang–Mills equations much further, replaced Euclidean space by a more general 4-dimensional manifold and proved highly unexpected results in differential topology that earned him a Fields Medal at the early age of 29.

It also had an influence on Michael's attitude to the interaction of mathematics and physics. Here was a piece of contemporary pure mathematics that provided a complete solution to a problem posed by physicists. Surely this could be taken further?

10. Mathematics and Physics

Michael's interaction with physicists had already begun before the work on instantons—one of his visits to MIT to see Singer had involved a discussion with R. Jackiw on anomalies, the Dirac operator and the index theorem. But after 1977 this took up a more substantial part of his life, much of which was involved with facilitating the interaction and educating physicists about concepts they weren't aware of (figure 8). Conversely, he also began to apply ideas from Yang–Mills theory to mathematical problems, perhaps the most notable his paper with Bott on Riemann surfaces and the Narasimhan–Seshadri theorem.

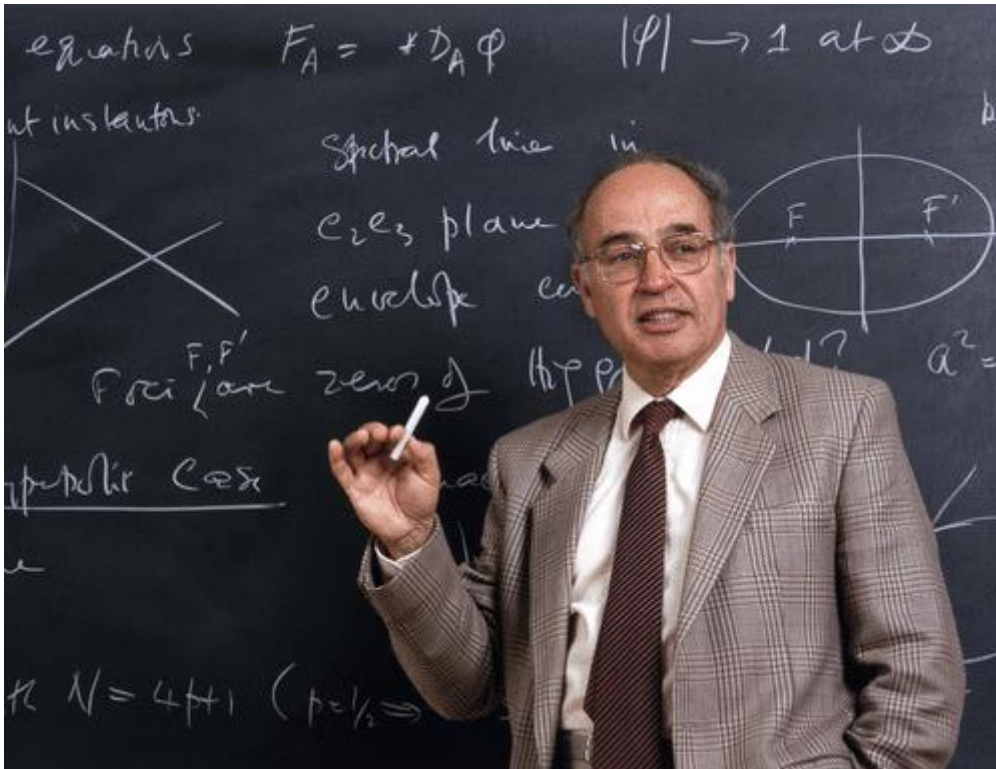


Figure 8. A lecture on monopoles. (Image: Corbin O'Grady Studio/Science Photo Library.) (Online version in colour.)

The most important go-between was the theoretical physicist Edward Witten (ForMemRS 1999). They first met in MIT in 1977 while Witten was a student, and an impressed Michael invited him to Oxford for a few weeks for discussions that were the beginning of a long association. Michael introduced Witten to some established pieces of mathematics that were unknown to the physics community—Morse theory, equivariant cohomology and moment maps. In turn, Witten reinterpreted these and fed back new insights to the mathematical community. Soon he was appointed as a permanent member at the Institute for Advanced Study and Michael's visits there were basically to see him.

In 1987–1988 Michael made several trips and in particular set Witten various challenges. One was seeing Donaldson theory in quantum-theoretic terms, which was achieved through a twisted version of supersymmetric Yang–Mills theory. Another was understanding the Jones (V. F. R. Jones, FRS 1990) polynomial of a knot via quantum field theory. This work brought into play a productive fusion of

ideas from different parts of mathematics and physics and contributed to Witten winning a Fields Medal in 1990. As Michael wrote of Witten in his citation for the International Congress: ‘Although he is definitely a physicist (as his list of publications clearly shows) his command of mathematics is rivalled by few mathematicians, and his ability to interpret physical ideas in mathematical form is quite unique. Time and again he has surprised the mathematical community by a brilliant application of physical insight leading to new and deep mathematical theorems.’ The concepts that arose out of this work, notably (building on ideas of Segal) that of a Topological Quantum Field Theory, formed the theme of Michael's Oxford Seminar for one term, the outcome being a published collection of stimulating ideas rather than a mathematical paper.

Michael had a remarkable intuition regarding the mathematics that was relevant to quantum field theory, and his rapport with Witten fed a two-way exchange of ideas—he had a reputation as a talker, but he was also a good listener. Yet he did not enter the physicist's territory to attempt to make quantum field theory mathematically rigorous. It was enough to see that the mathematical theory had a natural home there with the promise of new insights. Instead, the physics-oriented topics he wrote about concerned the classical rather than quantum aspects of gauge theories—the topology and geometry of moduli spaces of instantons and monopoles and the Skyrme model in nuclear physics. He collaborated, on the suggestion of his Trinity contemporary John Polkinghorne (soon to become an Anglican priest), with the Cambridge theoretical physicist Nicholas Manton (FRS 1996) on these themes, studying their geometry and dynamics as extended objects and not point particles.

If his actual work did not engage directly with the physicists' main aims, Michael nevertheless thought seriously about the relationship. In a Harvard conference for Bott's seventieth birthday in 1993, he outlined four reactions to the way in which mathematicians could interact with physicists:

- Try to give rigorous proofs of mathematical results predicted by the physicists.
- Try and understand the physics and enter into a dialogue using their language.
- Develop the physics on a rigorous basis with formal proofs.
- Understand the deeper meaning of the connection between mathematics and physics.

While accepting the validity of each approach, he admitted that he was attracted to the last one, even if it had an essentially philosophical content, raising the question of whether mathematics is created or discovered.

The dialogue continued even later. In 2002 he and Lily spent two winter months at CalTech, where Witten held a visiting position. Michael recalled: ‘During my stay I felt like a graduate student again, with Witten as my supervisor. It was intellectually demanding but very rewarding. It resulted in a mammoth joint paper in which I developed the details of the mathematics that Witten had assigned to me and he pursued the physics beyond my reach.’

11. Trinity College

Trinity College, Cambridge, was always a magnet for Michael. In 1983 he had hoped to become Master and when Andrew Huxley (PRS 1980–1985) was appointed, he thought his chance had gone—‘history only knocks once’. But in 1990 he was successful and left Oxford to move into the Master's Lodge.

At the same time, he became President of the Royal Society and founding director of the Isaac Newton Institute for Mathematical Sciences, a national facility based in Cambridge. This busy life, to and fro between Cambridge and London, left little time for mathematical research.

Michael had already had some experience of public service and learned societies at various stages of his career, having been involved in the Cockcroft Committee for the reform of mathematics teaching in schools, a president of both the London Mathematical Society and the Mathematical Association, a vice president of the Royal Society and active in the International Mathematical Union and in the formation of the European Mathematical Society.

Perhaps these activities, as well as his mathematical prowess, contributed to his knighthood, awarded in 1983. The two rather different audiences served by Trinity and the Royal Society entailed an extension of themes on which Michael was required to speak and write.

As for Trinity, he and Lily enjoyed entertaining both eminent visitors and students in the Lodge, although balancing their wishes for a relaxing home environment with college protocol took some time. One college Fellow recalls Lily as being the ‘last traditional Master's wife’. In fact in later years, Michael regretted that the conventions of the time had prevented Lily continuing an independent academic career, although she had been a very successful teacher at Headington School in Oxford.

He gave frequent addresses at the various college events and special occasions and even gave a sermon in the chapel (on Trinity Sunday 1997, which he hoped would become a convention). Michael was proud of Trinity's fame through the achievements of its alumni, and was quick to point out that some contemporary or historical personage ‘was a Trinity man’, which must have been the result of some considerable research while Master. On his watch, a Nobel Prize in economics was awarded to James Mirrlees, whom Michael had supervised when he was an undergraduate.

He did not arrive with any particular agenda for the college, but nevertheless some of his ideas were voted down. In particular it seems as if the Fellows did not wish to celebrate 450 years of existence and preferred to wait another 50 for a round number. In his retirement speech he offered a cynic's view of a Master's experience: ‘First you have to find out what the situation is: this takes several years. Next you have to formulate the changes you would like to see: this takes even longer.

Finally you realize it is impossible to make the changes: this you discover pretty quickly!’ It is not clear that he was entirely joking, but two changes did occur: the

research fellowships were opened up to non-Trinity applicants, and a portrait of one of Michael's scientific heroes, James Clerk Maxwell FRS, was hung in the College Hall. This was somewhat short of a statue next to Newton in the chapel, which he thought Maxwell deserved.

12. The Royal Society

Within a few weeks of being invited to be head of Trinity College, Michael was approached to be President of the Royal Society, which was somewhat more than he had expected (figure 9). His appointment conveniently broke the rigid A-side/B-side + Nobel Prize convention that adhered to boundaries which were changing in the scientific world.



Figure 9. President 1990–1995. (Image © Anne Purkiss, www.purkiss-images.eu.)

Soon after taking office, he made some organizational changes with fixed five-year terms for the officers, one falling vacant each year. This logical procedure also served as a creative solution to some sensitive internal issues. He also began a democratization process, with more openness in the election of members of council and officers. In particular, postal votes and contested elections for council members began.

The presidency necessitated speeches and meetings representing aspects of UK science to many different bodies, and allowed Michael to develop his own views that had been subdued in the years devoted to mathematics. External events, too, demanded responses, and one of these was the plight of scientists following the break-up of the Soviet Union. Research scientists were leaving the affected countries or being forced into unskilled jobs because of budget cuts. Michael was

well aware of the work his old friend Fritz Hirzebruch was doing in finding academic jobs for East German mathematicians in the same predicament. A report was sent to the UK prime minister and funding released to keep Russian science connected to the best work in the West.

One innovation introduced during this time was the establishment of the Academy of Medical Sciences in 1998, following the recommendations of a working group chaired by Michael. The founder president was Peter Lachmann FRS, the Society's Biological Secretary from 1993 to 1995, who first got to know Michael playing simultaneous chess games as a student in Trinity. He has acknowledged that Michael played 'a vital role in overcoming widespread opposition among medical organizations to the whole idea and persuaded many influential people of its merits, taking decisive action at a critical moment'. Michael's outward-looking convictions not only helped to bring together other national academies, such as the Royal Academy of Engineering, but also was instrumental in founding the Inter-Academy Panel, playing a similar international role.

As president, Michael found himself addressing academies, societies and institutes of all kinds, from the Caribbean academy to the Indian one, and from the issue of tobacco control to science in the National Portrait Gallery. However, there was one theme in the relationship between science and society that he returned to again and again: the threat posed by nuclear weapons. He reserved his final Anniversary Address, in 1995, 50 years after the dropping of the first atomic bomb, for an exposition of his personal views without the restraints of representing the Society. He contrasted the technical scientific accomplishment of nuclear weapons with the public suspicion of the scientist that followed.

The main thrust of his argument was that 'the insistence on a UK nuclear capability was fundamentally misguided, a total waste of resources and a significant factor in our relative economic decline over the past 50 years'. There was understandably significant press coverage, but in fact these were long-held views that he continued to expound when he became president of the Pugwash Movement in 1997, in succession to its founder, Joseph (later Sir Joseph) Rotblat (FRS 1995). Michael was particularly pleased when Rotblat was elected to the Royal Society and won the Nobel Peace Prize in the same year, 1995.

13. Edinburgh

On his retirement from Trinity College in 1997, Michael moved to Lily's home city of Edinburgh. They had frequently been up to Scotland to their log cabin in Kirkmichael, Perthshire, but now they bought a penthouse flat with views of both Arthur's Seat and the Pentland Hills, yet quite close to the mathematics department of the University of Edinburgh. The department provided an office that Michael used on a daily basis unless he was travelling. Elmer Rees, a former colleague and collaborator in Oxford, was a professor there, and there were younger members of staff with whom he could discuss ideas.

Here, after an absence of several years from research, Michael sought a problem to work on and found one thanks to a question from the physicist Michael Berry FRS arising from the spin statistics theorem. This was a simply stated problem about the existence of a continuous map with certain properties from the configuration space of n points in three-dimensional Euclidean space to the flag manifold $U(n)/T$. He devoted six papers to this theme. A resolution and generalization appeared in a joint paper with Roger Bielawski, but this depended on gauge-theoretic ideas—somehow this was not to Michael ‘a proof consistent with the elegance of the problem’ and he was never quite satisfied with it.

Michael's subsequent work again had an interface with physics, as the string theorists had realized that D-brane charges were best represented as K -theory classes, and this led to a return to his old subject and the new twisted versions of it. But another facet began, the change from ‘inadvertent physicist’ to ‘intentional physicist’ in the words of Bernd Schroers, his collaborator at Heriot-Watt University.

At this stage Michael wanted to attack physical problems head-on but with unorthodox ideas, avoiding quantum theory, which he disliked as being too linear despite its obvious achievements. These proposals were highly speculative, one involving advanced and retarded terms in the equations, another based on so-called ‘geometric models of matter’—four-dimensional objects where topological invariants were supposed to correspond to baryon and lepton numbers and charges. These could have been viewed as metaphors, but Michael was convinced that they could provide new physical insights. Schroers (Hitchin 2019) describes this period: ‘He generated new ideas at a prodigious rate, announcing them with infectious enthusiasm, but abandoning them casually if a new and more promising avenue presented itself.’ Towards the end of his life, this enthusiasm carried him perhaps a bridge too far as he attempted to resolve long-standing questions in mathematics, but he carried on working until the end.

If these eccentric approaches to theoretical physics did not attract followers, there was one event in his final year that would vindicate his search for topological ideas in real physics. The 2019 Breakthrough Prize in Physics was awarded to C. Kane and E. Mele for their work on topological insulators, a new kind of material. The theoretical prediction of their existence, soon experimentally verified, appeared in 2019 (Kane & Mele 2005), where we may read: ‘These bundles are classified

within the mathematical framework of twisted real K -theory ... the Z_2 index is related to the mod 2 index of the real Dirac operator.'

Mathematics was not Michael's only interest in his newly adopted home. The Royal Society of Edinburgh became a substitute for Trinity College, and he would enjoy entertaining guests there. He became more and more involved with the Society and appreciated the fact that it covered the humanities as well as science. Finally, he became its president from 2005 until 2008. His time at the Royal Society of London served as 'a good apprenticeship', as he commented when elected to the position. Michael became quite well known for his contributions to public life in Scotland, felt at home having had a Scottish mother and wife, and increasingly preferred the political complexion of the devolved government as opposed to the Conservatives in Westminster. He became well-informed about the Scottish Enlightenment and became more likely to emphasize the Scottish dimension of famous men rather than their attachment to Trinity College. Most importantly, his mission to erect a statue of Maxwell, denied in Trinity, was successful in Edinburgh and it stands prominently in George Street, close to Maxwell's original home (figure 10).



Figure 10. With Dr Alasdair Allan MSP, the Minister for Learning, Science and Scotland's Language, in front of the James Clerk Maxwell statue in George Street, Edinburgh in 2015. (Image kindly supplied by the Royal Society of Edinburgh).

14. Later Life

Apart from integrating into the social and academic life of Edinburgh, there were several other activities that Michael engaged in after leaving Cambridge. There was the Order of Merit, traditionally awarded to retiring Presidents of the Royal Society. He enjoyed the regular lunches with the Queen and other members of the Order. There were the Chancellorship of the University of Leicester from 1995 to 2012, numerous honorary degrees and foreign membership of academies providing yet more scope for speeches. One activity he particularly enjoyed was the Heidelberg Laureates Forum, where he could talk with enthusiasm and encouragement to young scientists, offering advice and inspiration. He was a wonderful raconteur at this stage, regaling young and old with stories, stopping just short of indiscretion. Despite occasional health issues, Michael continued to travel abroad and he addressed the last International Congress of Mathematicians in Rio de Janeiro just a few months before he died.

There was also a return to his roots in the Middle East. The civil war in Lebanon ended in 1990 and Michael went back at regular intervals. The American University of Beirut, founded in 1866, was the premier university in the region for many years. Since 1887 the language of instruction was English and it was there that Michael's grandfather studied medicine in the 1890s, introducing the English language into the family. Several cousins also taught there in the twentieth century, and gradually Michael became more involved. Eventually the Center for Advanced Mathematics was set up with Michael as chairman of its advisory board. Later, the Simons Foundation made a donation of two million dollars to establish a chair in his name. Continuing this association, the London Mathematical Society has recently set up the Atiyah UK–Lebanon Fellowship scheme in his memory.

Lily Atiyah died on 13 March 2018 at the age of 90, Michael on 11 January 2019. He is survived by two sons, David and Robin. His eldest son, John, died in 2002 while on a walking holiday in the Pyrenees.

15. Awards and recognition

- 1962 Fellow of the Royal Society
- 1966 Fields Medal
- 1968 Royal Medal
- 1980 De Morgan Medal, London Mathematical Society
- 1981 Antonio Feltrinelli Prize, Accademia dei Lincei
- 1983 Knight Bachelor
- 1985 Fellow, Royal Society of Edinburgh
- 1987 King Faisal Prize
- 1988 Copley Medal
- 1992 Order of Merit
- 1993 Benjamin Franklin Medal, American Philosophical Society
- 1993 Nehru Centenary Medal, Indian National Science

Academy

- 1993 Honorary Fellow, Royal Academy of Engineering
- 1993 Commander of the Order of the Cedars, Lebanon
- 1996 Freedom of the City of London
- 1997 Order of Andres Bello, Venezuela
- 1999 Honorary Fellow, Faculty of Actuaries
- 2001 Honorary Fellow, Academy of Medical Sciences
- 2004 Abel Prize
- 2005 Order of Merit (Gold) Lebanon
- 2008 President's Medal, Institute of Physics
- 2010 Grande Médaille, French Academy of Sciences

2010 National Order of
Scientific Merit of Brazil

2011 Grand Officier Légion
d'honneur

2012 Fellow, American
Mathematical Society

Foreign Membership

1969 American Academy of Arts
and Sciences

1972 Royal Swedish Academy

1977 Akademie Leopoldina

1978 US National Academy of
Sciences

1978 Académie des Sciences

1979 Royal Irish Academy

1983 Third World Academy of
Science

1992 Australian Academy of
Science

1992 Ukrainian Academy of

Science

- 1993 Indian National Science Academy
- 1994 Russian Academy of Science
- 1996 Georgian Academy of Science
- 1998 American Philosophical Society
- 1997 Venezuelan Academy of Science
- 1999 Accademia dei Lincei
- 1999 Chinese Academy of Science
- 2001 Czechoslovakia Union of Mathematics
- 2001 Norwegian Academy of Science and Letters
- 2001 Moscow Mathematical Society
- 2002 Royal Spanish Academy of

Science

2008 Lebanese Academy of
Sciences

Honorary degrees

1968 Bonn University

1969 University of Warwick

1979 University of Durham

1981 University of St Andrews

1983 University of Chicago

1983 Trinity College, Dublin

1984 University of Cambridge

1984 University of Edinburgh

1985 University of London

1985 University of Essex

1987 University of Ghent

1990 University of Reading

1990 University of Helsinki

1991 University of Leicester

1992 Rutgers University

1992 University of Salamanca

1993 University of Montreal

1993 University of Wales

1993 University of Waterloo

1994 Lebanese University

1994 Queen's University,
Kingston, Canada

1994 University of Keele

1994 University of Birmingham

1995 Open University

1996 UMIST

1996 Chinese University of
Hong Kong

1997 Brown University

1998 University of Oxford

1998 Charles University, Prague

1999 Heriot-Watt University

2001 National University of Mexico

2004 American University of Beirut

2005 University of York

2006 Harvard University

2007 Scuola Normale, Pisa

2008 Polytechnic University of Catalonia

16. Acknowledgments

A prime source for this memoir was an extended series of interviews, with myself as facilitator, which took place while Michael was Master of Trinity; these are available online. They also formed the basis of a biography in Yau (2003), wherein other recollections can be found. Memories from a more recent period are in Hitchin (2019). The collected works contain Michael's comments focused on the background to the mathematical papers. I also wish to thank David Atiyah and participants at the two memorial events in Edinburgh and Cambridge for several enlightening contributions.

Figures [1](#) to [3](#) were kindly provided by the Atiyah family. The portrait photograph was taken in 1985 by Godfrey Argent, © Godfrey Argent Studio.

17. Author profile



Nigel Hitchin, the author of this biography, studied mathematics in Oxford, gaining a DPhil in 1972. He worked with Michael Atiyah in Princeton and Oxford, in particular on various aspects of gauge theory in pure mathematics. After 1990 he held chairs at Warwick and Cambridge before returning to Oxford as Savilian Professor of Geometry in 1997. His work links together differential geometry, algebraic geometry and the equations of theoretical physics. Since 2016 he has been an emeritus professor.

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